Cosmology and the origin of structure

Rocky I: The universe observed

Rocky II: The growth of cosmological structure

Rocky III: Inflation and the origin of perturbations

Rocky IV: Dark matter and dark energy

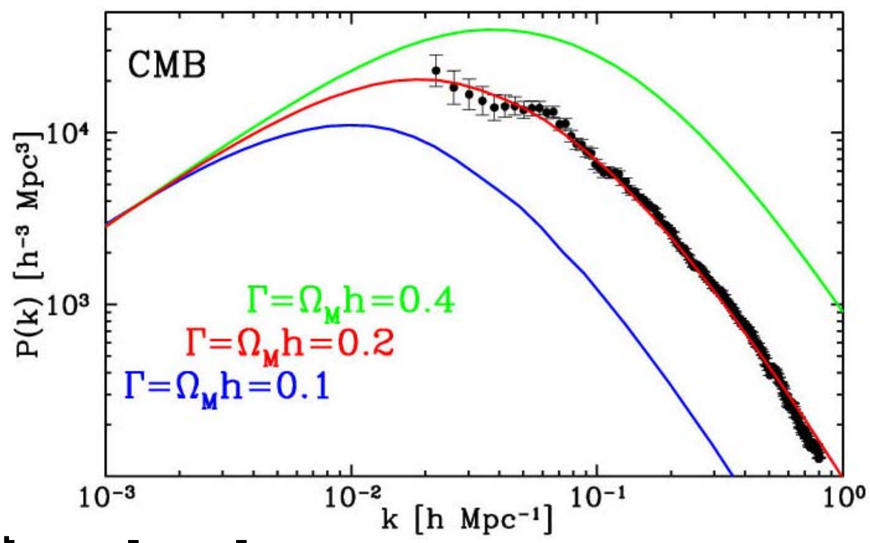
Academic Training Lectures

Rocky Kolb

Fermilab, University of Chicago, & CERN

Rocky II: Growth of structure

- Linear regime: quantative analysis
  - Jeans analysis
  - Sub-Hubble-radius perturbations (Newtonian)
  - Super-Hubble-radius perturbations (GR)
  - Harrison-Zel'dovich spectrum
  - Dissipative processes
  - The transfer function
  - Linear evolution



- Non-linear regime: word calculus
  - Comparison to observations
  - A few clouds on the horizon

Growth of small perturbations

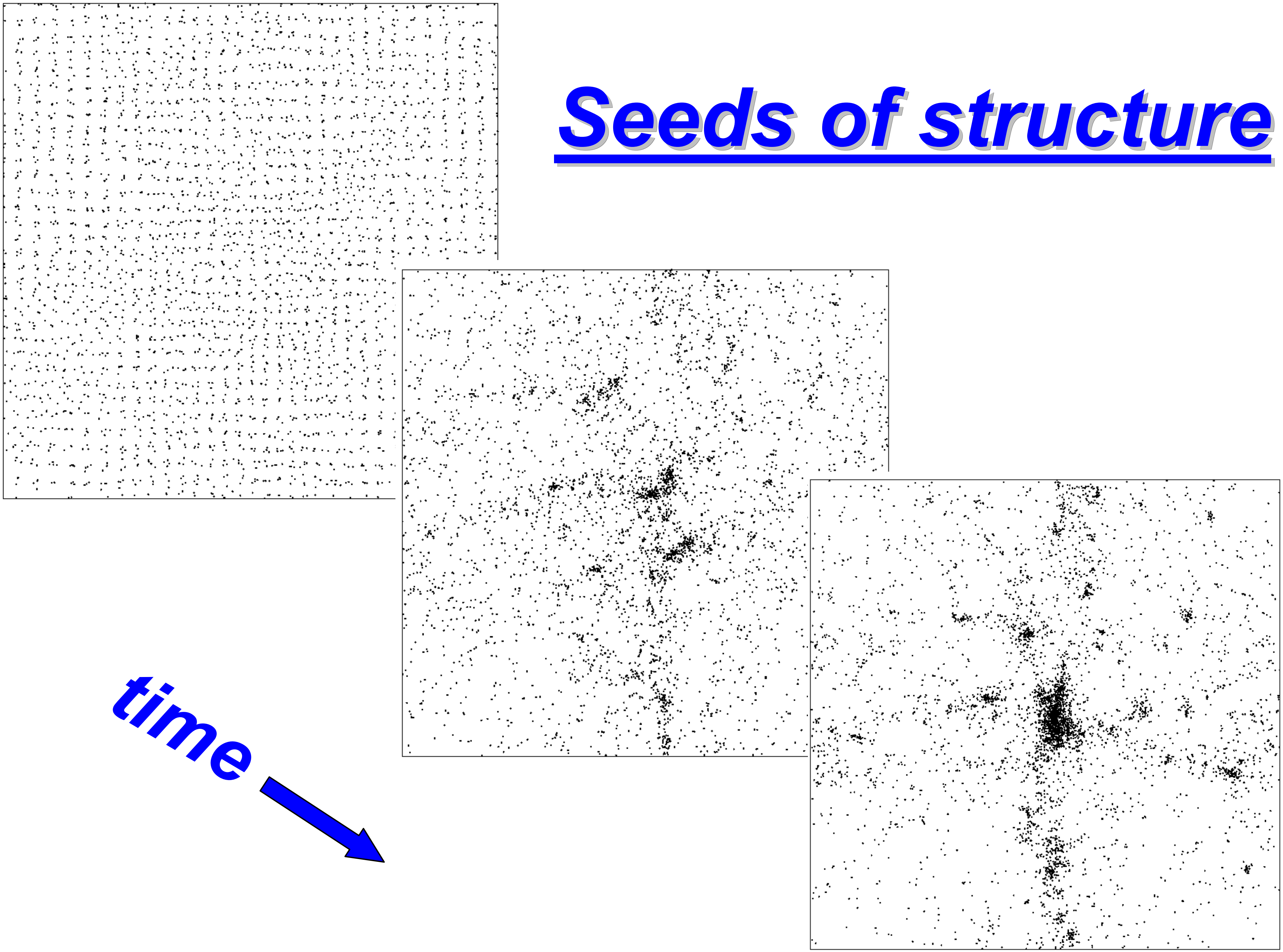
Today (12 Gyr AB)

- radiation and matter decoupled
- $\Delta T / T \sim 10^{-5}$
- $\Delta\rho_G / \rho_G \sim 10^{+6}$

Before recombination (300 kyr AB)

- radiation and matter decoupled
- $\Delta T / T \sim 10^{-5}$
- $\Delta\rho_G / \rho_G \sim 10^{-5}$

Seeds of structure



Simulation

Simulation (simiŭlēi·ʃən). ME. [a. OF., ad. L. *simulationem*.] **1.** The action or practice of simulating, with intent to deceive; false pretence, deceitful profession ME. **b.** Unconscious imitation 1870. **2.** A false assumption or display, a surface resemblance or imitation, *of* something.

Oxford English Dictionary

Power spectrum

- Assume there is an average density $\bar{\rho}$
- Expand density contrast $\delta\left(\vec{x}\right)$ in Fourier modes

$$\delta\left(\vec{x}\right) \equiv \frac{\rho\left(\vec{x}\right) - \bar{\rho}}{\bar{\rho}} = \int \delta_{\vec{k}} \exp\left(-i\vec{k}\cdot\vec{x}\right) d^3k$$

- Autocorrelation function defines power spectrum

$$\left\langle \frac{\delta\rho(\vec{x})}{\rho} \right\rangle^2 = \left\langle \delta(\vec{x})\delta(\vec{x}) \right\rangle = \int_0^\infty \frac{dk}{k} \; \frac{k^3 \left|\delta_{\vec{k}}^2\right|}{2\pi^2}$$

$$\Delta^2(k) \equiv \frac{k^3 \left|\delta_{\vec{k}}^2\right|}{2\pi^2} \qquad\qquad P(k) \equiv \left|\delta_{\vec{k}}^2\right|$$

Jeans analysis

Jeans analysis in a non-expanding fluid:

matter density ρ

pressure p

velocity field $\vec{v}$

gravitational potential ϕ

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho \vec{v} \right) = 0 \\ \dfrac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} + \dfrac{1}{\rho} \vec{\nabla} p + \vec{\nabla} \phi = 0 \\ \nabla^2 \phi = 4\pi G \rho \end{cases}$$

Perturb about solution*

$$\rho = \rho_0 = \text{constant} \qquad \rho = \rho_0 + \rho_1$$

$$p = p_0 = \text{constant} \qquad p = p_0 + p_1$$

$$\vec{v} = 0 = \text{constant} \qquad \vec{v} = \vec{v}_0 + \vec{v}_1$$

$$\phi = \phi_0 = \text{constant} \qquad \phi = \phi_0 + \phi_1$$

$$\boxed{\frac{\partial^2 \rho_1}{\partial t^2} - v_s^2 \nabla^2 \rho_1 = 4\pi G \rho_1}$$

$$v_s^2 = p_1 / \rho_1$$

Jeans analysis

$$\boxed{\frac{\partial^2 \rho_1}{\partial t^2} - v_s^2 \nabla^2 \rho_1 = 4\pi G \rho_1}$$

solutions of the form $\rho_1(\vec{r},t) = \delta(\vec{r},t)\rho_0$

$$= A_k \exp\left(-i\vec{k}\cdot\vec{r} + i\omega t\right)$$

ω and k satisfy the dispersion relation $\omega^2 = v_s^2 k^2 - 4\pi G \rho_0$

ω real: perturbations oscillate as sound waves

ω imaginary: exponentially growing (or decaying) modes

Jeans wavenumber

$$k_J = \left(\frac{4\pi G \rho_0}{v_s^2}\right)^{1/2}$$

$k > k_J$ perturbation oscillates

$k < k_J$ perturbation grows

Jeans mass

$$M_J = \frac{4\pi}{3}\left(\frac{\pi}{k_J}\right)^3 \rho_0$$

$M < M_J$ perturbation oscillates

$M > M_J$ perturbation grows

gravitational pressure vs. thermal pressure

Sub-Hubble-radius ($R_H = H^{-1}$)

Jeans analysis in an expanding fluid: scale factor *a(t)* describes expansion, unperturbed solution:*

$$\rho_0 = \rho_0(t_0)\, a^{-3}(t) \qquad \vec{v}_0 = \frac{\dot{a}}{a}\vec{r} \qquad \vec{\nabla}\phi_0 = \frac{4\pi G \rho_0}{3}\vec{r}$$

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k + \left(\frac{v_s^2 k^2}{a^2} - 4\pi G \rho_0\right)\delta_k = 0$$

- Solution is some sort of Bessel function: growth or oscillation depends on Jeans criterion
- In matter-dominated era $\rho_0 = \left(6\pi G t^2\right)^{-1}$ and $\dot{a}/a = 2/3t$
- For wavenumbers less than Jeans

$$\delta_+(t) = \delta_+(t_i)\left(t/t_i\right)^{2/3} \qquad \delta_-(t) = \delta_-(t_i)\left(t/t_i\right)^{-1}$$

Super-Hubble-radius $(R_H = H^{-1})$

$$g_{\mu\upsilon}(\vec{x},t) = g_{\mu\upsilon}^{FRW}(t) + \delta g_{\mu\upsilon}(\vec{x},t)$$

$$T_{\mu\upsilon}(\vec{x},t) = T_{\mu\upsilon}^{FRW}(t) + \delta T_{\mu\upsilon}(\vec{x},t)$$

$$\delta R_{\mu\upsilon} - (1/2)\,\delta\left[g_{\mu\upsilon} R\right] = 8\pi G \delta T_{\mu\upsilon}$$

- complete analysis not for the faint of heart
- interested in "scalar" perturbations
- fourth-order differential equation
- only two solutions "physical"
- other two solutions are "gauge modes" which can be removed by a coordinate transformation on the unperturbed metric

$$\boxed{\delta R_{\mu\upsilon} - (1/2)\,\delta\left[g_{\mu\upsilon} R\right] = 8\pi G \delta T_{\mu\upsilon}}$$

Bardeen 1980

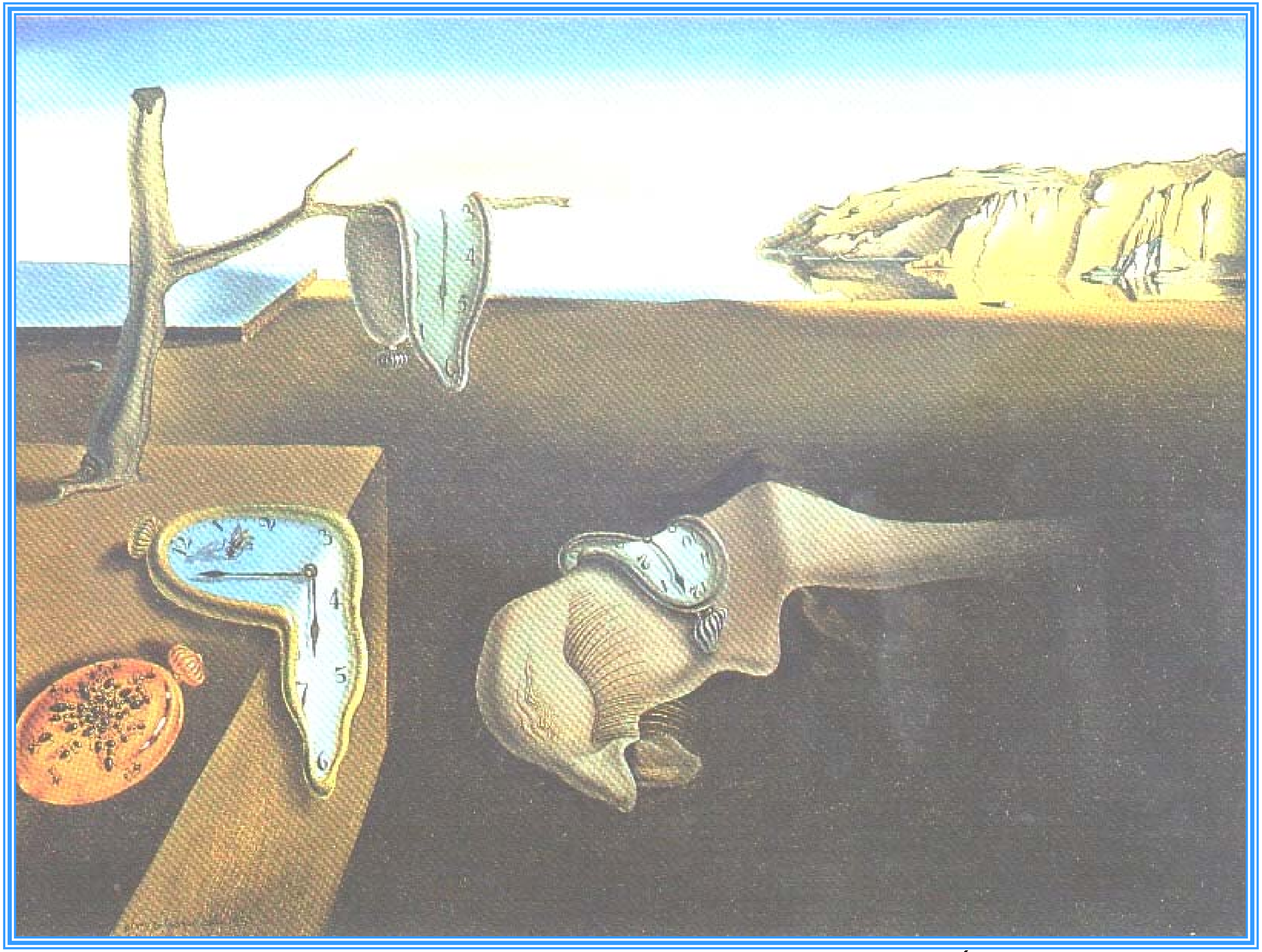
Reference spacetime: flat FRW

$$ds^2 = a^2(\eta)\left\{ d\eta^2 - \delta_{ij}\, dx^i dx^j \right\}$$

$$\eta = \text{conformal time}$$

$$dt^2 = a^2(\eta)\, d\eta^2$$

η : conformal time



$$dt^2 = a^2(\eta)\, d\eta^2 \qquad ds^2 = a^2(\eta)\left(d\eta^2 - d\vec{x}^{\,2}\right)$$

$$\boxed{\delta R_{\mu\upsilon} - (1/2)\,\delta\left[g_{\mu\upsilon} R\right] = 8\pi G\,\delta T_{\mu\upsilon}}$$

Bardeen 1980

Reference spacetime: flat FRW

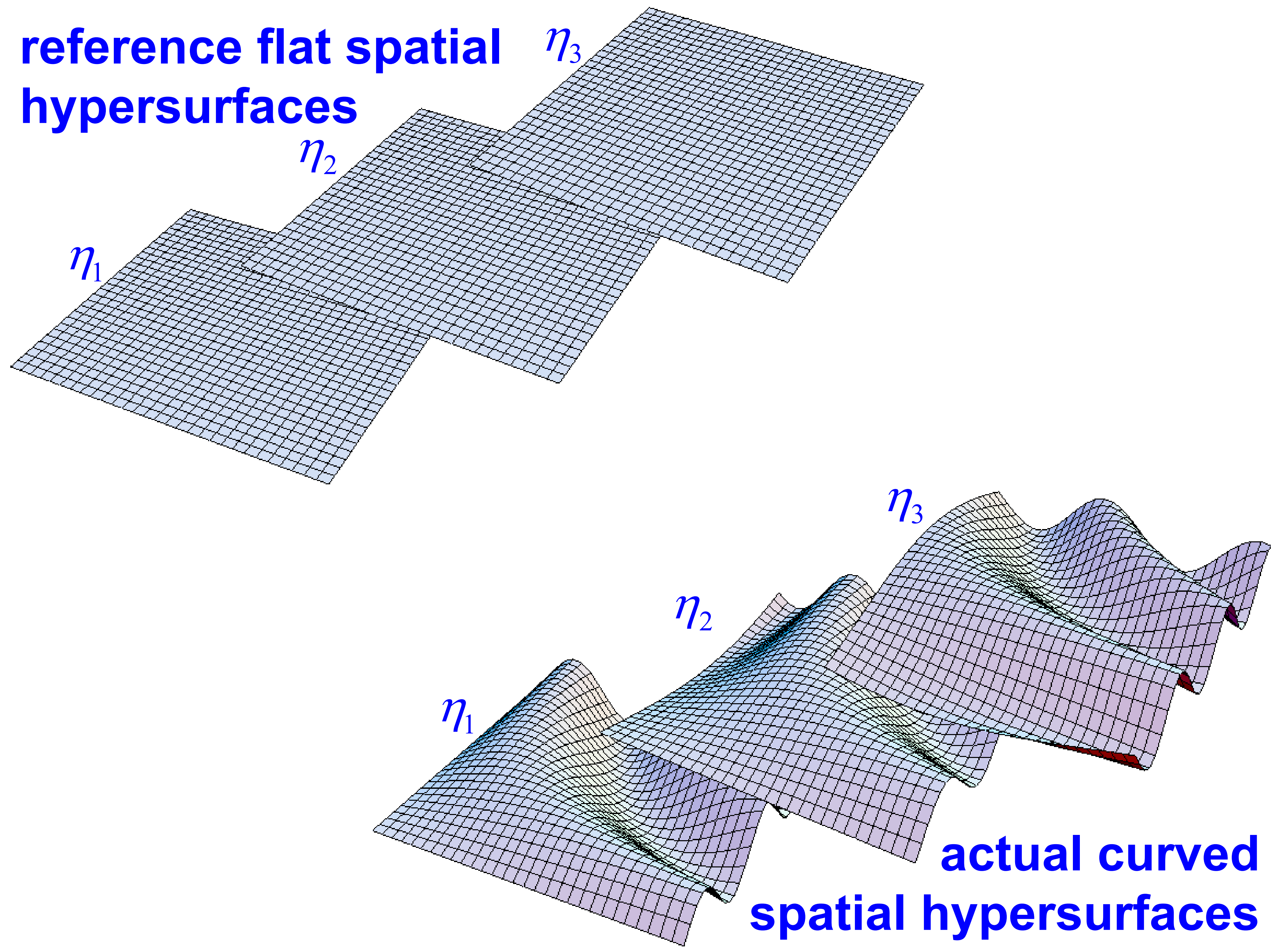
$$ds^2 = a^2(\eta)\left\{d\eta^2 - \delta_{ij}\,dx^i dx^j\right\}$$

$$\eta = \text{conformal time}$$

$$dt^2 = a^2(\eta)\,d\eta^2$$

Perturbed spacetime (10 degrees of freedom):

$$ds^2 = a^2(\eta)\left\{\left(1 + \delta g_{00}\right)d\eta^2 - 2\delta g_{0i}\,d\eta\,dx^i - \left(\delta_{ij} + 2\delta g_{ij}\right)dx^i dx^j\right\}$$


reference flat spatial hypersurfaces
η_3
η_2
η_1
η_3
η_2
η_1
actual curved spatial hypersurfaces

scalar, vector, tensor decomposition

$$\delta g_{00} = 2A \qquad \qquad 1$$

$$\delta g_{0i} = S_i + \partial_i B \qquad \qquad 2+1$$

$$(\partial^i S_i = 0)$$

$$\delta g_{ij} = h_{ij} - \psi \delta_{ij} + \partial_i F_j + \partial_j F_i + \partial_i \partial_j E \qquad \qquad \underline{2+1+2+1}$$

$$(h^i{}_i = 0 \;\; ; \;\; \partial^i h_{ij} = 0 \;\; ; \;\; \partial^i F_i = 0) \qquad \qquad \boxed{10}$$

evolution of scalar, vector, and tensor perturbations decoupled

Vector Perturbations:

- are not sourced by stress tensor
- decay rapidly in expansion

Tensor Perturbations:

- perturbations of transverse, traceless component of the metric: gravitational waves
- do not couple to stress tensor

Scalar Perturbations

- couple to stress tensor
- *density perturbations!*

Super-Hubble-radius

in synchronous gauge $A = B = 0$

and uniform Hubble flow gauge $B = E = 0$

$$\delta_+(t) = \delta_+(t_i)\left(t/t_i\right)^{2/3} \qquad \text{matter-dominated}$$

$$\delta_+(t) = \delta_+(t_i)\left(t/t_i\right) \qquad \text{radiation-dominated}$$

- in matter-dominated era

$$\delta_+(t) = \delta_+(t_i)\left(t/t_i\right)^{2/3} \qquad \text{scales larger than Hubble radius}$$

$$\delta_+(t) = \delta_+(t_i)\left(t/t_i\right)^{2/3} \qquad \text{scales smaller than Hubble radius}$$

- in radiation-dominated era

$$\delta_+(t) = \delta_+(t_i)\left(t/t_i\right) \qquad \text{scales larger than Hubble radius}$$

$$\delta_+(t) = \text{constant} \qquad \text{scales smaller than Hubble radius}$$

Harrison-Zel'dovich

in radiation-dominated era

$$\delta_+(t) = \delta_+(t_i)\left(t/t_i\right) \qquad \text{scales larger than Hubble radius}$$

$$\delta_+(t) = \text{constant} \qquad \text{scales smaller than Hubble radius}$$

in matter-dominated era

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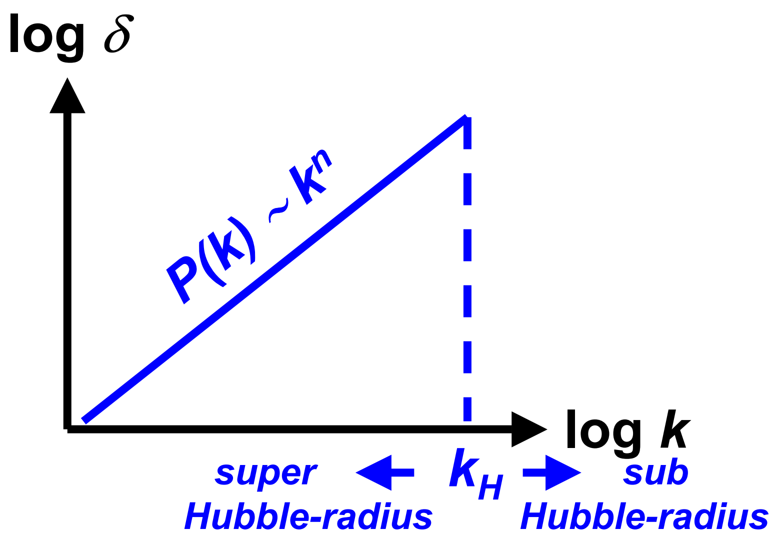
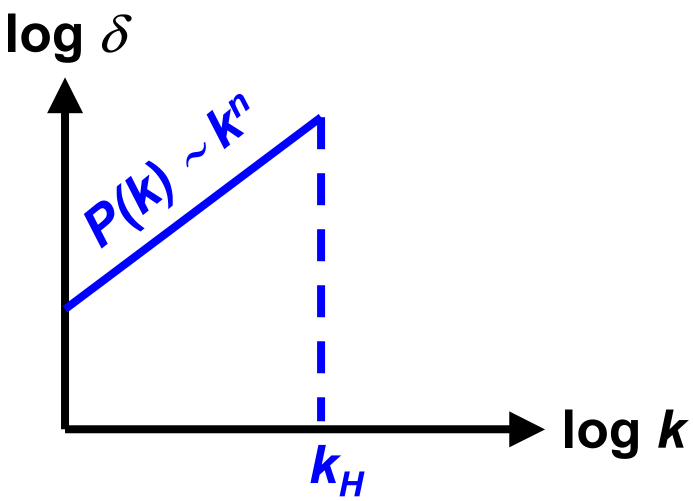
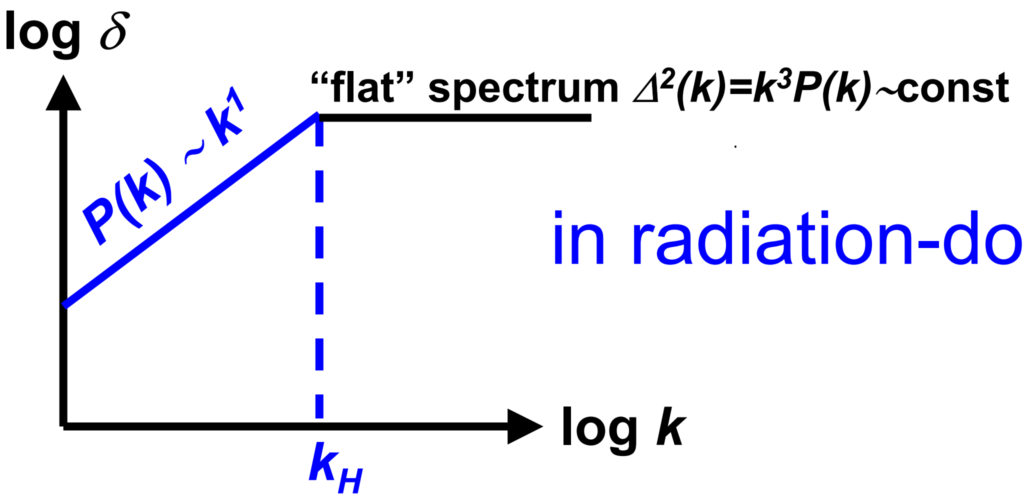




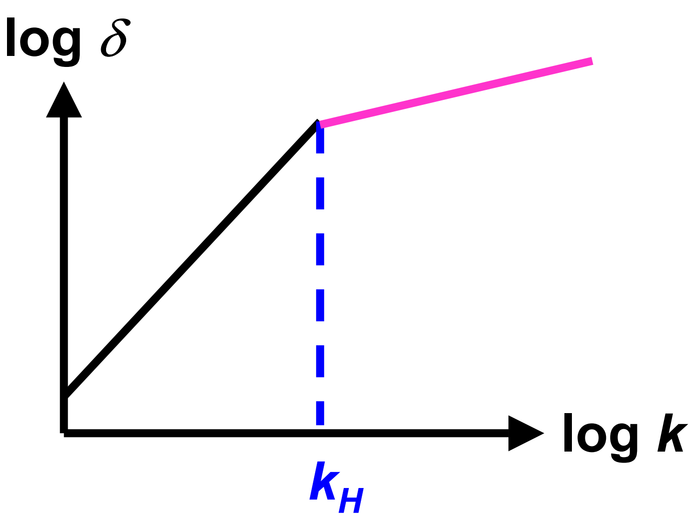
Harrison-Zel'dovich



in radiation-dominated era

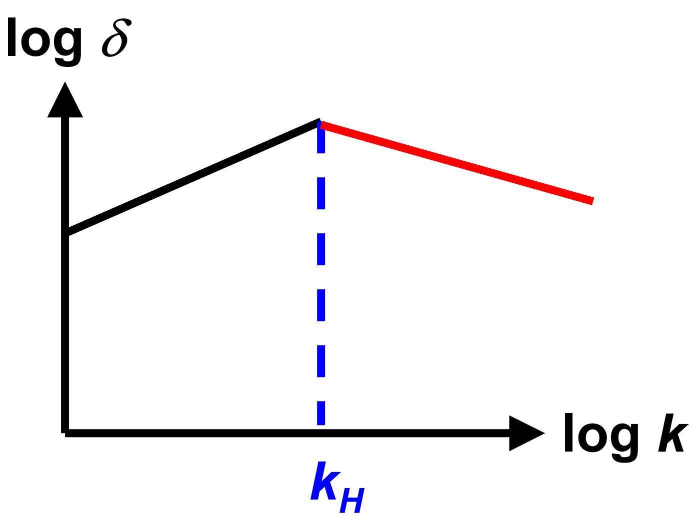
$$P(k) \propto k^n \quad n > 1$$

ultraviolet catastrophe

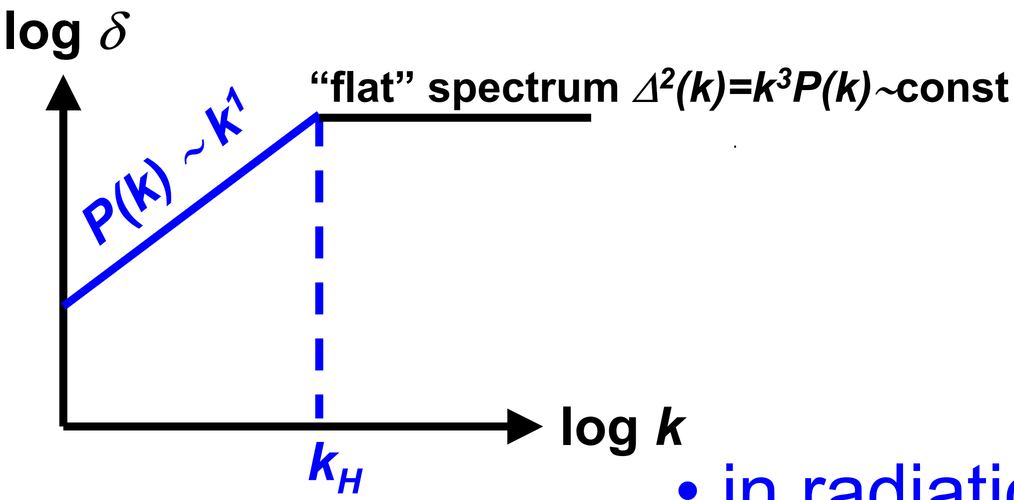


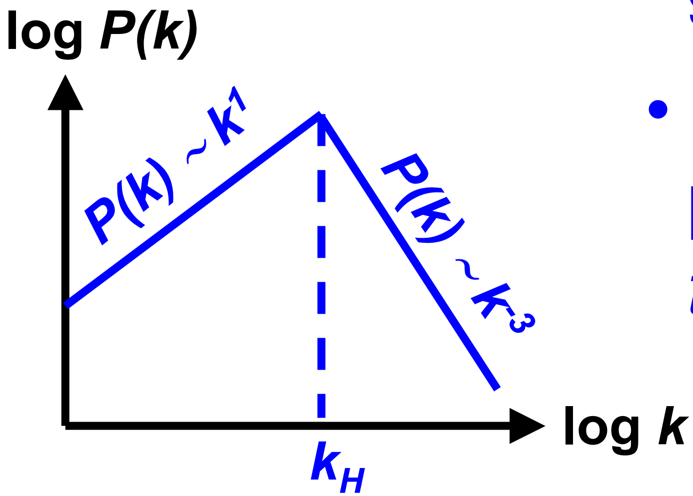
$$P(k) \propto k^n \quad n < 1$$

infrared catastrophe



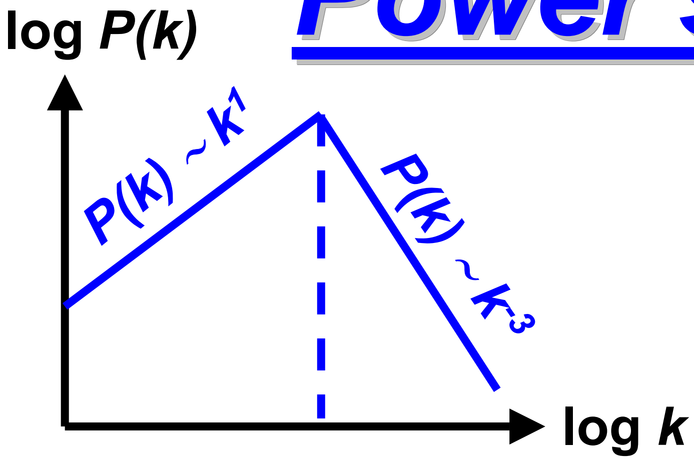
Harrison-Zel'dovich

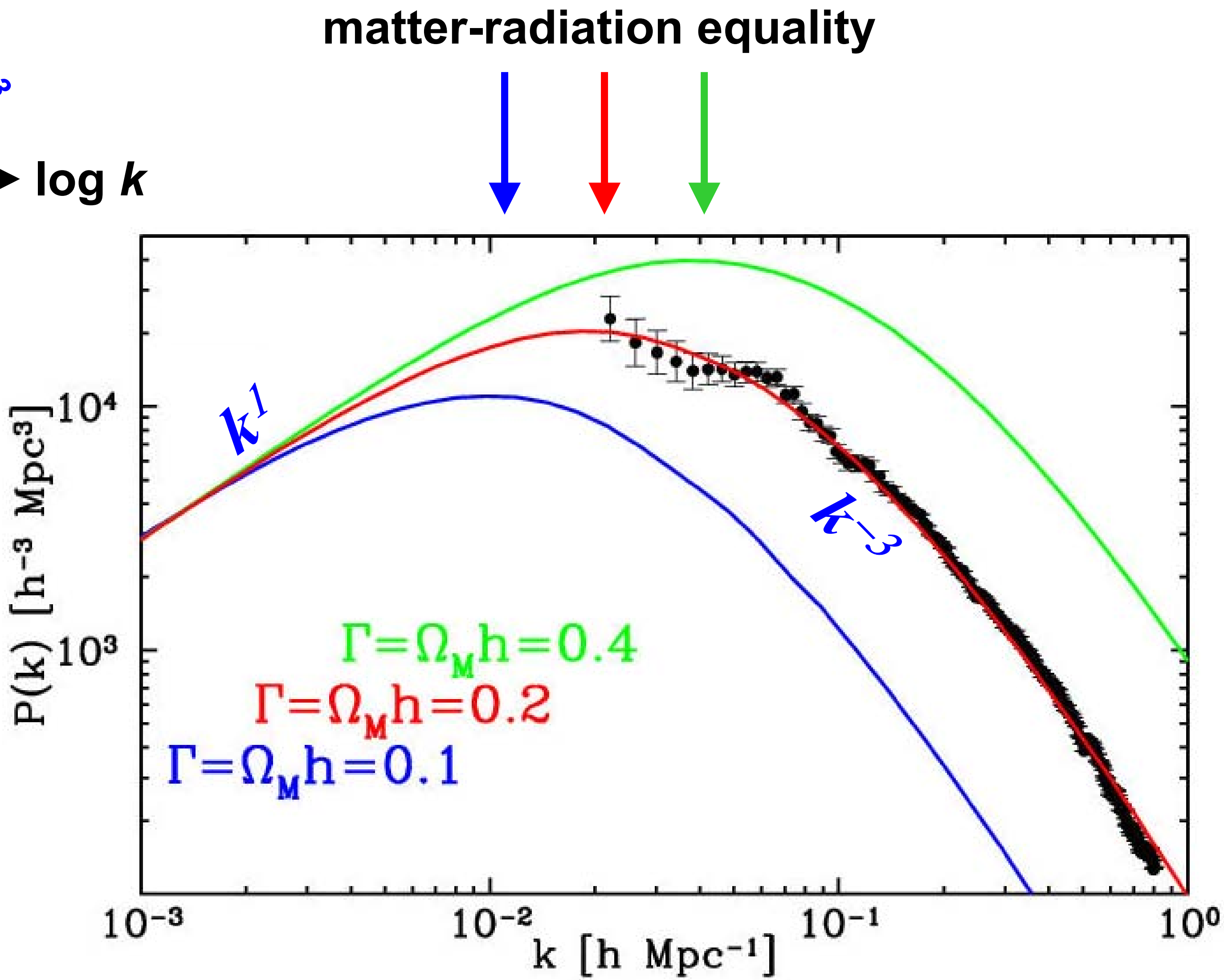




- in radiation-dominated era
  no growth sub-Hubble radius
  growth as t super-Hubble radius
- in matter-dominated era
  power spectrum grows as
  $t^{2/3}$ on all scales

Power spectrum for CDM





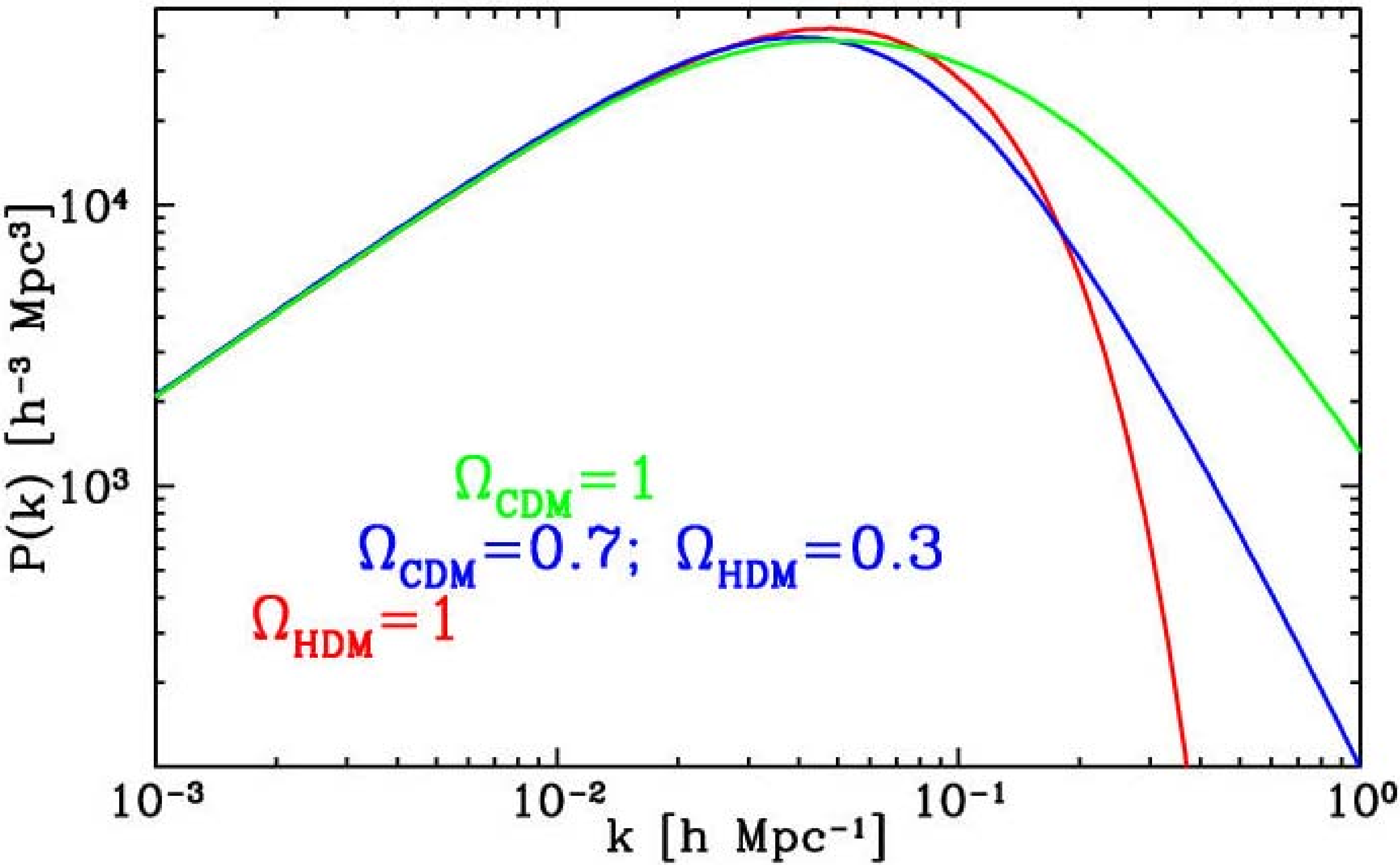
Dissipative processes

1. Collisionless phase mixing – free streaming

If dark matter is relativistic or semi-relativistic particles can stream out of overdense regions and smooth out inhomogeneities. The faster the particle the longer its free-streaming length.

Quintessential example: eV-range neutrinos

The evolved spectrum



Dissipative processes

1. Collisionless phase mixing – free streaming

If dark matter is relativistic or semi-relativistic particles can stream out of overdense regions and smooth out inhomogeneities. The faster the particle the longer its free-streaming length.

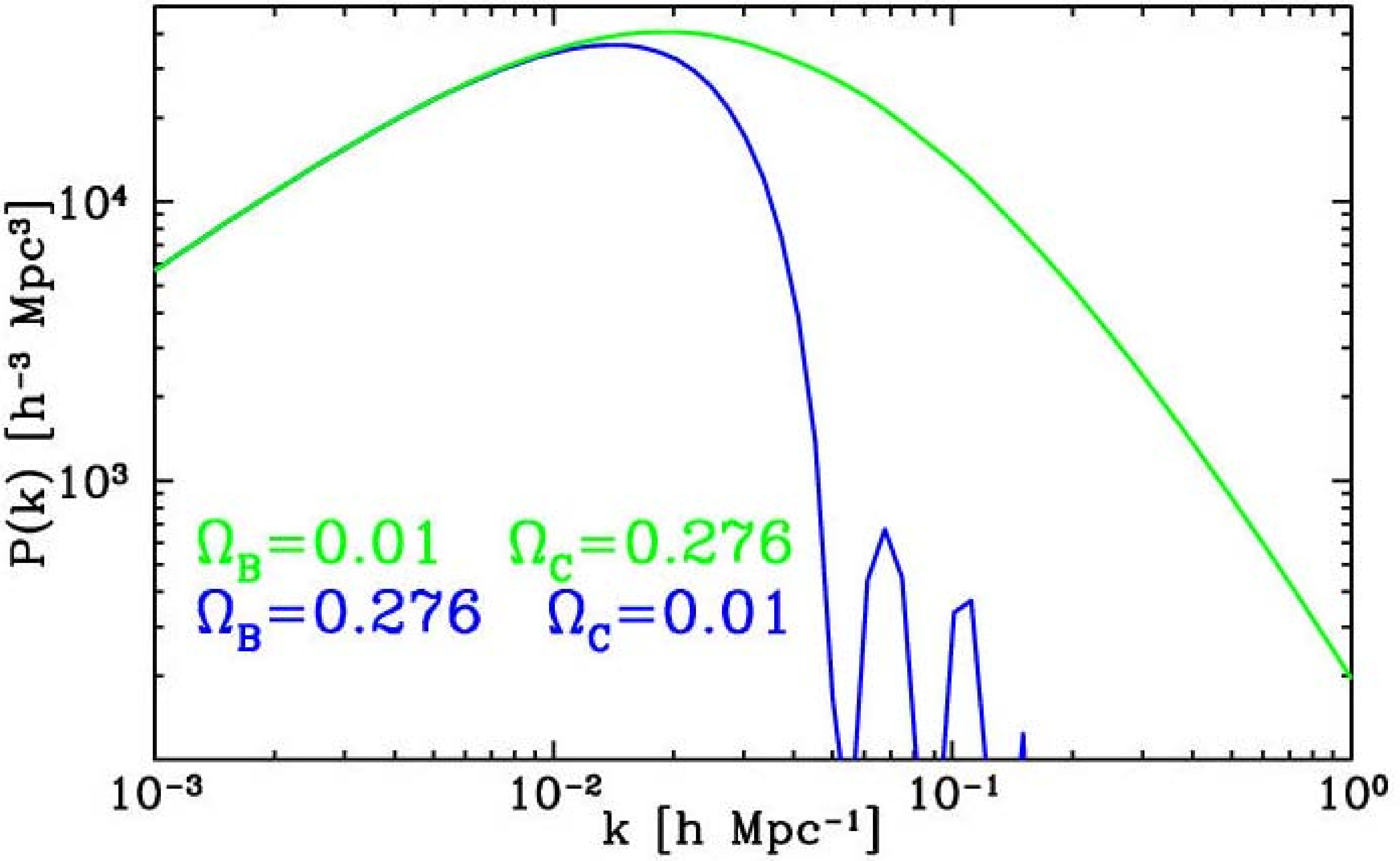
Quintessential example: eV-range neutrinos

2. Collisional damping – Silk damping

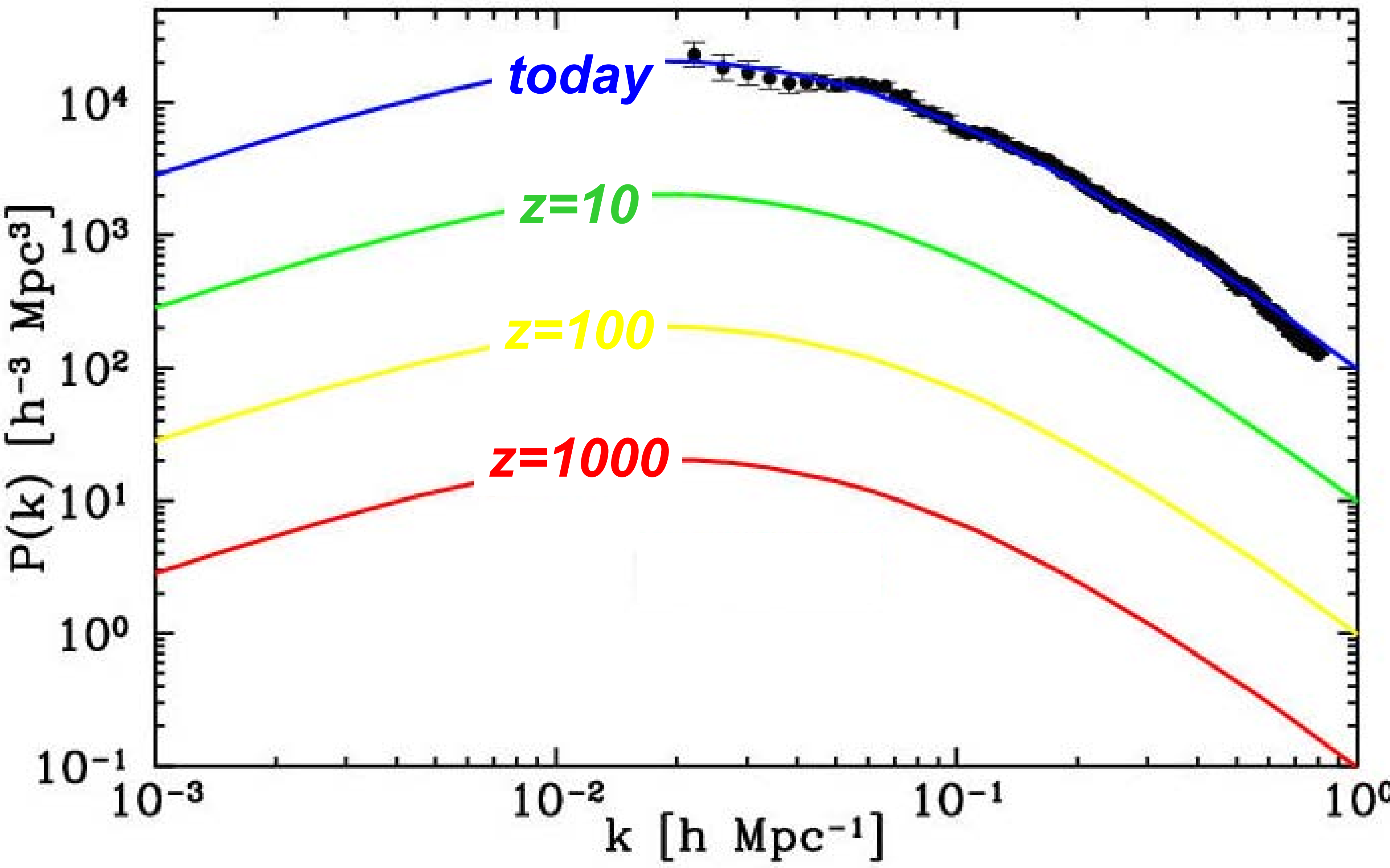
As baryons decouple from photons, the photonmean-free path becomes large. As photons escape from dense regions, they can drag baryons along, erasing baryon perturbations on small scales.

Baryon-photon fluid suffers damped oscillations.

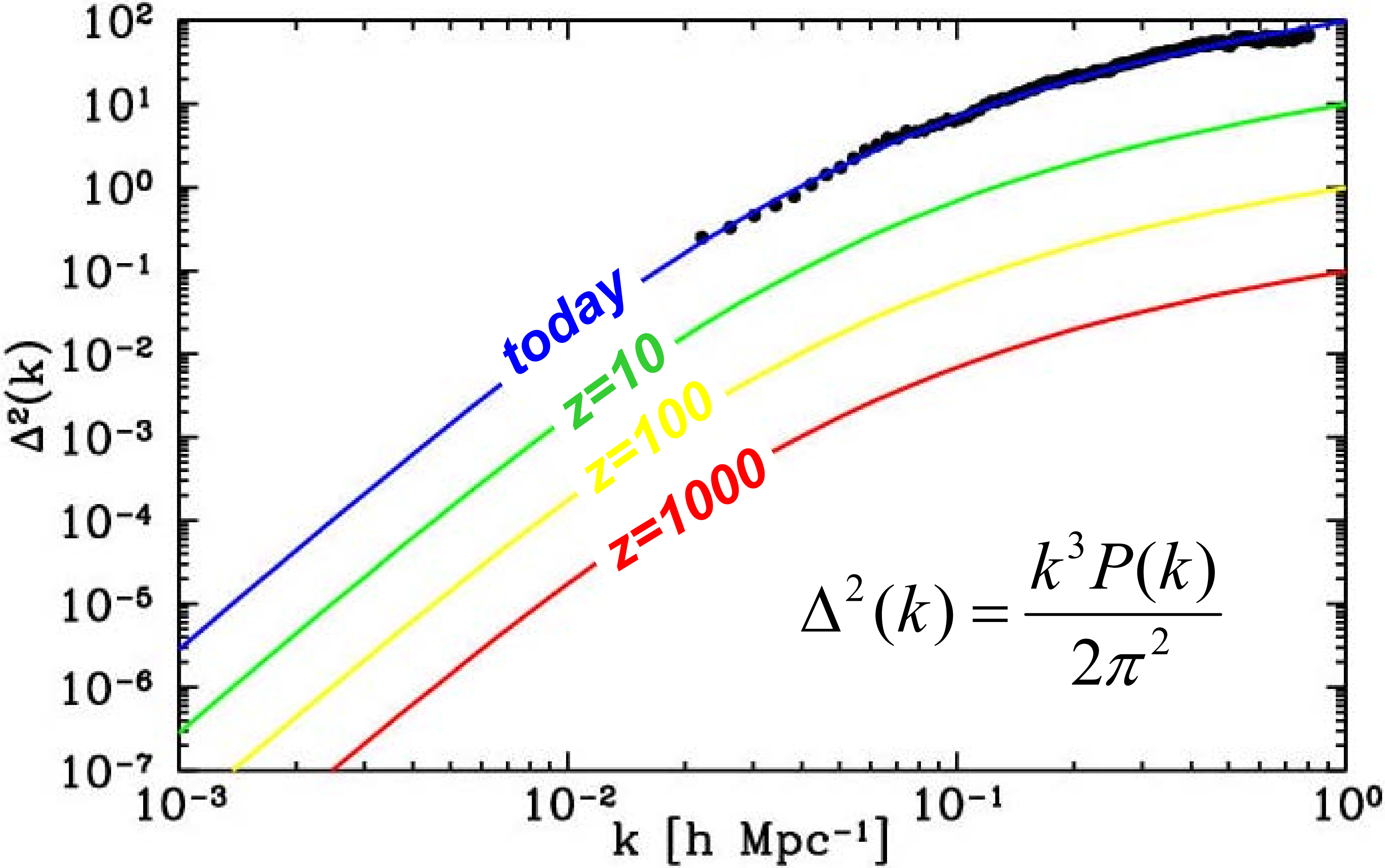
The evolved spectrum



Linear evolution

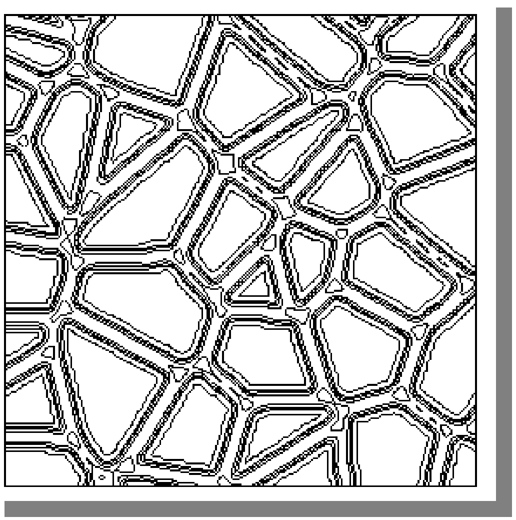


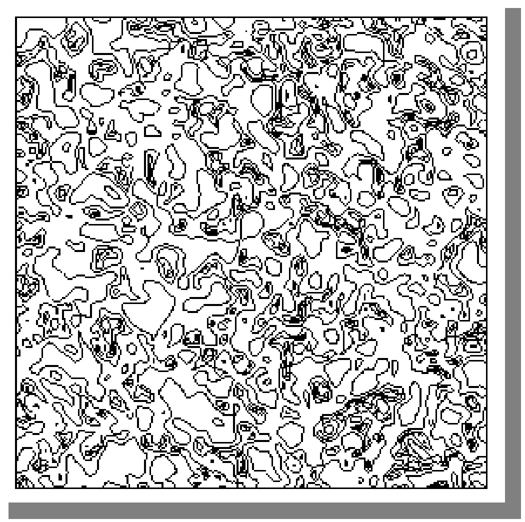
Linear evolution



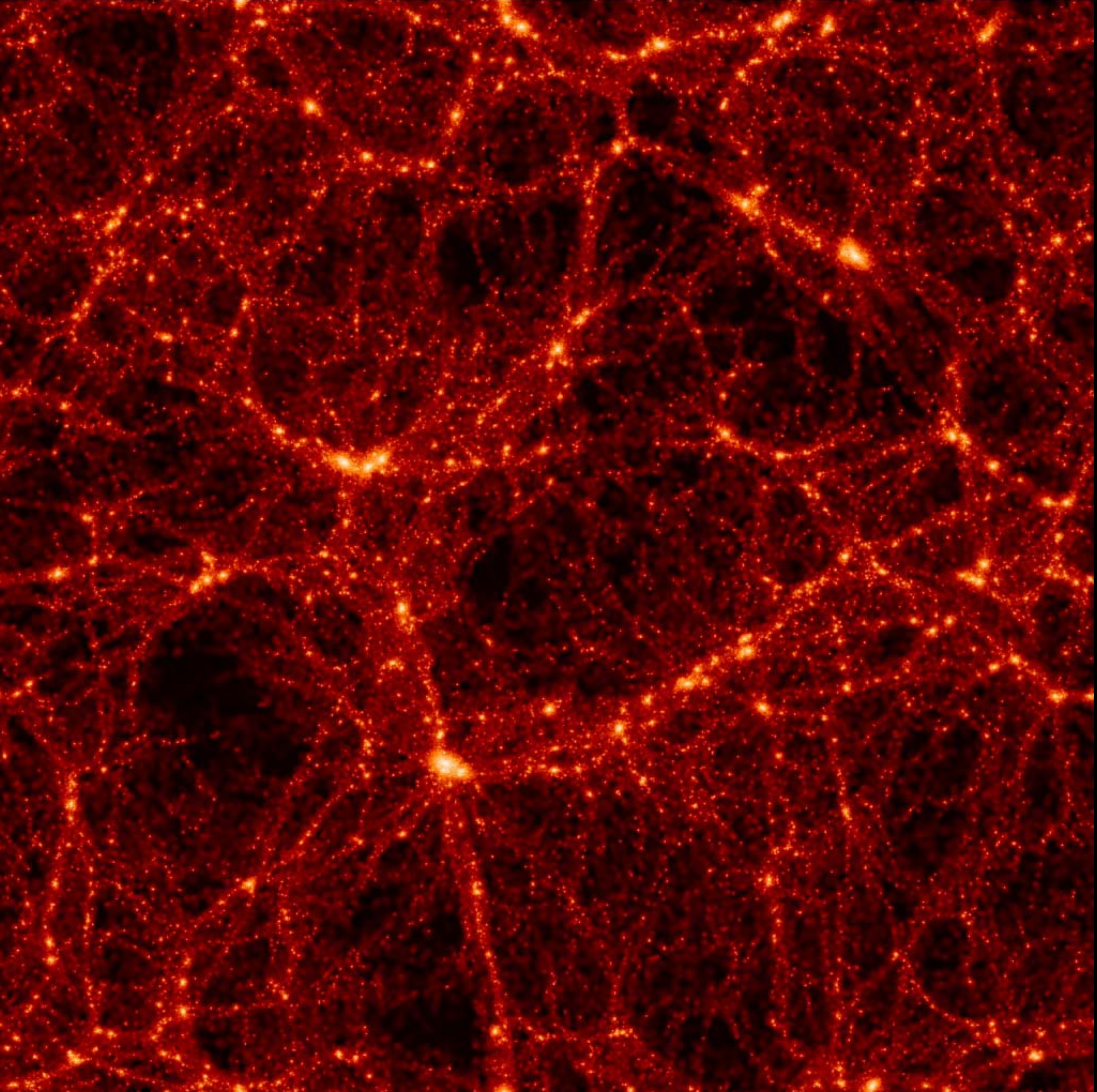
Life ain't linear!

- Many scales become nonlinear at about the same time
- Mergers from many smaller objects while larger scales form
- N-body simulations for dissipation-less dark matter
- Hydro needed for baryons
- Power spectrum well fit if $\Gamma = \Omega h \sim 0.2$
- There is more to life than the power spectrum



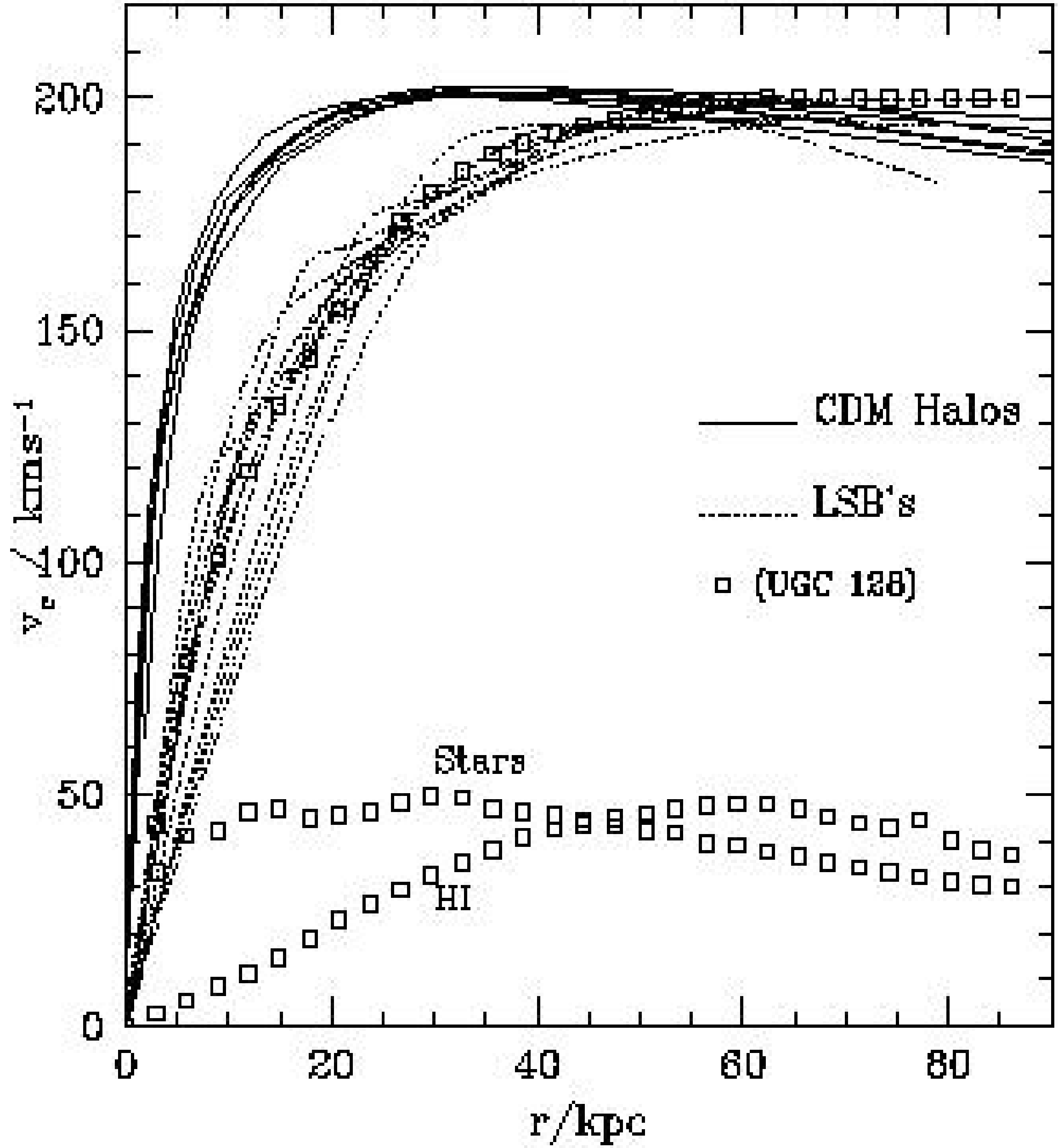


Alex Szalay



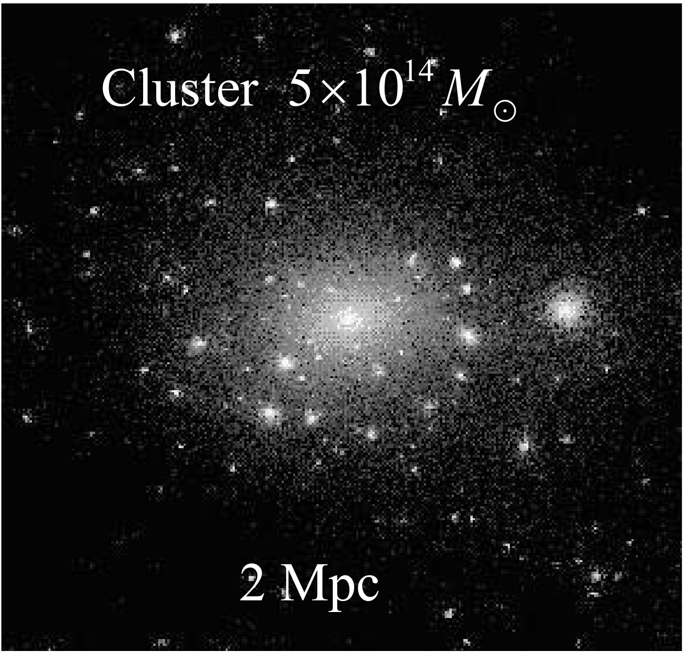
Large-scale structure fits well

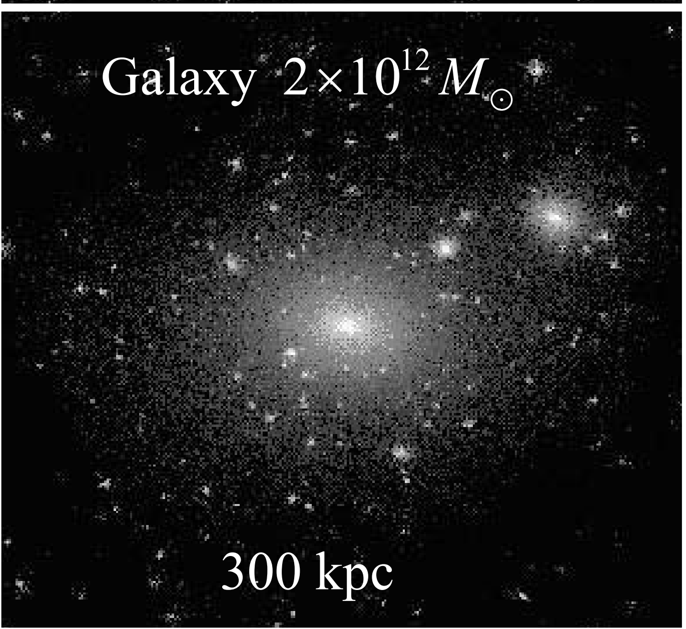
Small-scale structure-cusps

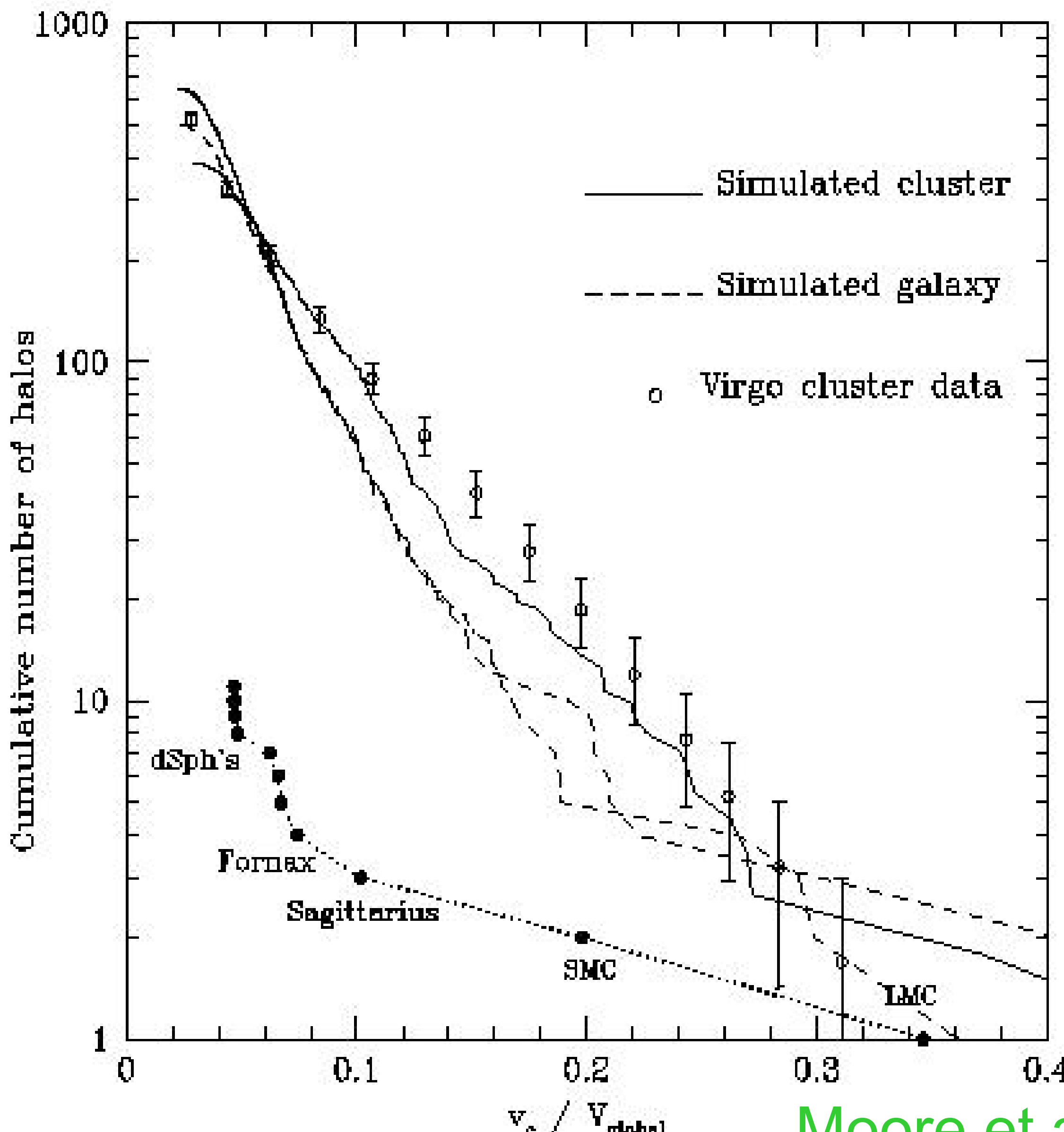


Moore et al.

Small-scale structure-satellites







Moore et al.

Rocky *II*: Growth of structure

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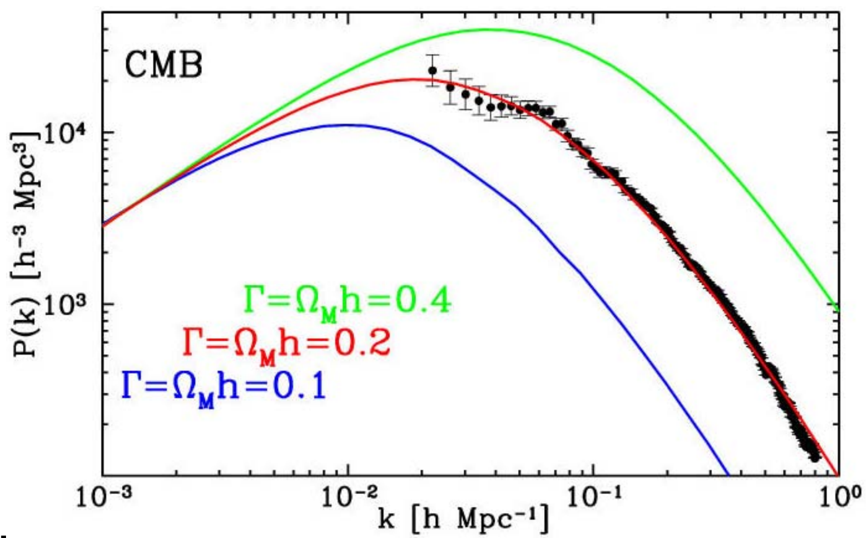

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